

# Zeno Quantum Gates in Semiconductor Quantum Dots

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## Abstract

We propose a scheme for a two-qubit conditional phase gate by quantum Zeno effect with semiconductor quantum dots. The system consists of two charged dots and one ancillary dot that can perform Rabi oscillations under a resonant laser pulse. The quantum Zeno effect is induced by phonon-assisted exciton relaxation between the ancillary dot and the charged dots, which is equivalent to a continuous measurement. We solve analytically the master equation and simulate the dynamics of the system using a realistic set of parameters. In contrast to standard schemes, larger phonon relaxation rates increase the fidelity of the operations.

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The Quantum Zeno effect (QZE) [1, 2, 3, 4] occurs when a rapid sequence of measurements is performed on a slowly evolving quantum system, with the result that the system is *frozen* in its initial state. An analogous effect occurs when a system is strongly coupled to a reservoir, as the transfer of information from the system into the reservoir mimics a continuous measurement. If coupling to the reservoir can be made contingent on the joint quantum state of two qubits, the QZE can be used in conjunction with control pulses to efficiently drive the qubits into an entangled state. This approach have been discussed within the framework of interaction free measurements [5, 6, 7], decoherence free subspaces [8, 9], as well as counterfactual quantum computation [10]. Proposed physical implementations vary from purely photonic systems [11, 12, 13], to atom-cavity systems [7, 14], and superconducting qubits [15].

Following a generalized QZE phase-gate described in [7], we have devised a two-qubit conditional phase gate using electron spins in semiconductor quantum dots. This system has the advantage that decoherence rates are of the order of picoseconds, which can lead to significant improvements in gate time and/or fidelity relative to atomic systems. We consider a system composed of three quantum dots (QDs), two of which are singly charged with electrons. The spins of these two electrons are then the logical qubits on which the phase-gate acts. A laser field is then applied, tuned to the exciton resonance of the uncharged dot. The energy levels and laser polarization are chosen so that an exciton in the neutral dot can relax to the neighboring dots by a spin-conserving dissipative phonon-assisted process in which the exciton is annihilated into a virtual photon and recreated as an exciton and a phonon in a neighboring dot [16]. The emission of a phonon clearly carries away the information that at least one qubit spin did not match the electron spin of the exciton, otherwise the decay mechanism would have been Pauli blocked. Thus the possibility of phonon emission is equivalent to a continuous partial measurement [7, 17] of the collective spin state of the two qubits.

Despite the widely-held belief that decoherence must always be minimized in quantum information processing, it has been known for some time [14] that decoherence can in principle be harnessed to generate high-fidelity entanglement by use of the QZE. In our scheme, the QZE effect occurs when the strong dissipation rate of the exciton state suppresses the laser induced Rabi oscillations in the neutral dot, effectively freezing it in its ground state. As the dissipation mechanism is subject to Pauli blocking, the spin-qubits in the charged

electrons can thus be seen as *quantum switches*. If both switches are set to *off* (all electron spins aligned up) then the measuring device (the environment) is inactivated and the QZE does not occur. In this way, the driving laser can create a tri-partite entangled state involving the two spin qubits and the state of the neutral dot.

In order to predict the fidelity of state formation, we will use parameters appropriate for vertically grown (In,As)Ga/GaAs self-assembled quantum dots. Structures with vertically coupled neutral and charged quantum dots have been recently demonstrated [18]. Henceforth, the central neutral dot will be QD1, and the two lateral charged dots will be referred to as QD2 and QD3. Absorption of a photon creates an exciton state in QD1, and trion states in QD2 and QD3. We assume that the ground trion energies in the two lateral dots are similar, and are lower than the ground exciton energy in the central dot, so that phonon-mediated exciton relaxation is energetically allowed. In the absence of the QZE, the driving laser will induce Rabi Oscillations between the zero- and one-exciton states of QD1. Assuming that the driving laser field is  $\sigma_-$  polarized, the standard selection rules lead to the creation of an exciton with electron spin up( $+\frac{1}{2}$ ) and a heavy hole spin down( $-\frac{3}{2}$ ) in QD1. Due to the difference between the exciton and trion energy, QD2 and QD3 are far-detuned and thus not driven by the laser.

There are several exciton decay mechanisms that can spoil the Rabi oscillations. Many such processes depend on the intensity of the laser and have been experimentally characterized [19, 20]. In the weak excitation limit, phonon-mediated processes are dominant. The role of the phonon is to carry away excess energy. As phonon-assisted relaxation of a *single* carrier between two dots via tunneling is exponentially suppressed for QD separations of several nm, we focus on the aforementioned *exciton* relaxation via long-range phonon-assisted excitation transfer [16]. For a weak resonant  $2\pi$  laser pulse with Rabi energy  $\Omega \ll \Gamma$ , where  $\Gamma$  is the phonon emission rate, then a single spin-down electron will be sufficient to impose the QZE and freeze the system. When both spins are up, the  $2\pi$  pulse will give an overall  $\pi$  phase to the wave-function of the system. A schematic view of the different possible QZE scenarios for different initial states is shown in Fig.1. In any case, the final state of the ancillary system will be  $|0\rangle_a$ , so that the  $\pi$  phase shift is effectively imprinted only on the  $|\uparrow\uparrow\rangle_{23}$  state, thus realizing a two-qubit phase gate.

For concreteness, we will assume that the phonon-assisted excitation transfer is the dominant dissipation channel from QD1 to QD2 or QD3. However, the scheme for the gate and

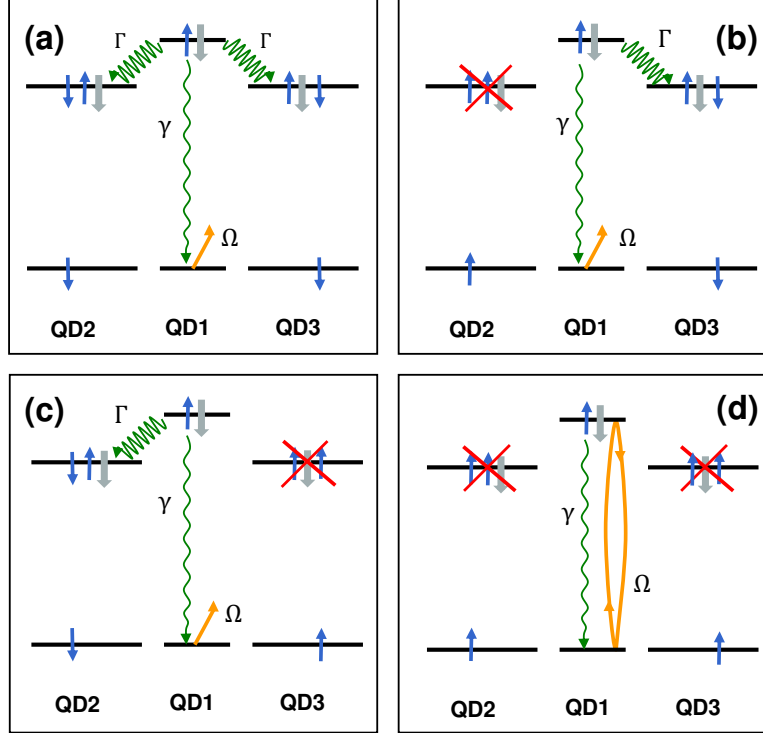


FIG. 1: (Color online) Scheme of the dynamics of the system under different initial states. The narrow blue (bold gray) arrow represents the electron (hole) spins. The energy levels for QD1 are the empty dot (lower) and the first exciton level (upper), while the energy levels in QD2 and QD3 are charged dot ground states (lower) and trion states (upper). Figures (a)-(d) correspond to the four possible initial states of the two qubits. (a)-(c) If either electron in QD2 and QD3 is spin down, the exciton in QD1 can decay into the neighboring dots, in which case the QZE prevents the Rabi Oscillation in QD1. (d) If both decay channels are closed, QD1 will undergo a  $2\pi$ -Rabi oscillation. The photon emission rate  $\gamma$  is too weak to induce a QZE.

our analytical results, can be easily adapted to the case in which QDs are close and the phonon emission involves only the electron. In this short-range case the energy levels of the central and lateral dots have to be engineered so that hole transfer is forbidden.

The quantum state of the system can be expressed in the triple-particle basis  $|\lambda\sigma\sigma'\rangle = |\lambda\rangle_a \otimes |\sigma\sigma'\rangle_{23}$ . Here,  $|\lambda\rangle_a$  ( $\lambda = 0, 1, 2, 3$ ) represents the state of the ancillary electron-hole pair created in QD1, where  $\lambda = 0$  is the vacuum state with no exciton, while  $\lambda = 1, 2, 3$  indicates the exciton residing in QD1, QD2, or QD3, respectively.  $|\sigma\sigma'\rangle_{23}$  represents the combined state of the two logical qubits, with  $\sigma, \sigma' = \uparrow, \downarrow$  indicating the spin (up or down) states of the electrons in QD2 and QD3, respectively. We note that during the Zeno gate operation, the probability to excite more than one exciton is exponentially small, and hence only the single-excitation case is considered in the present discussion. The states  $|2\uparrow\sigma'\rangle$  and  $|3\sigma\uparrow\rangle$  are forbidden by the Pauli exclusion principle, and therefore excluded from our model.

Our goal is to realize a two-qubit phase gate for the electron spins in QD2 and QD3, with the electron-hole pair acting only as an ancillary system. Ideally, such a gate transforms an initial logical state  $|\Psi_i\rangle_{23}$  of QD2 and QD3 to the final state  $\hat{U}_\pi|\Psi_i\rangle_{23}$ , with the  $\pi$ -phase gate operator defined via  $\hat{U}_\pi|\sigma\sigma'\rangle_{23} = (1 - 2\delta_{\sigma,\uparrow}\delta_{\sigma',\uparrow})|\sigma\sigma'\rangle_{23}$ . The ancillary system, initially prepared in  $|0\rangle_a$  state, becomes entangled with the logical qubits during the  $2\pi$  pulse, becoming once again disentangled by the end of the pulse. In practice, the ancillary qubit could still be entangled with the logical qubits after the gate operation, so that the final density matrix representing QD2 and QD3 is obtained by tracing over the  $|\lambda\rangle_a$  states.

The system's Hamiltonian is given by

$$H = \epsilon_2 c_2^\dagger c_2 + \epsilon_3 c_3^\dagger c_3 + \frac{\Omega}{2}(c_1 + c_1^\dagger) \quad (1)$$

Here, the  $c_i^\dagger(c_i)$  is the exciton creation (annihilation) operator, with  $i = 1, 2, 3$  labeling the three QDs.  $\Omega$  is the Rabi strength of the driving laser. In contrast to the state-selectivity of the phonon-mediated relaxation process, the decay of the exciton in QD1 via spontaneous photon emission is independent of QD2 and QD3 states, and will only cause the exciton to relax back to the initial  $|0\rangle_a$  state. This is the primary source of error in the gate operation, and is mitigated by choosing  $\Omega \gg \gamma$ , where  $\gamma$  is the exciton spontaneous photon-emission rate.

To model the system's dynamics, we employ the standard Linblad formalism [21] to arrive

at the master equation

$$i\frac{\partial\rho}{\partial t} = -[\rho, H] + i\mathcal{L}[\rho], \quad (2)$$

where  $\rho$  is the density operator for the system. The superoperator  $\mathcal{L}$  is given by

$$\mathcal{L}[\rho] = \frac{1}{2}[L_\gamma\rho L_\gamma^\dagger + L_\Gamma\rho L_\Gamma^\dagger - (L_\gamma^\dagger L_\gamma + L_\Gamma^\dagger L_\Gamma)\rho + H.c.], \quad (3)$$

where  $L_\gamma = \sqrt{\gamma}c_1$  describes spontaneous photon decay in QD1, and  $L_\Gamma = \sqrt{\Gamma}(c_2^\dagger c_1 + c_3^\dagger c_1)$  describes phonon-assisted dissipation from QD1 to QD2 and QD3. We note that aside from the photon- and phonon- relaxation channels, other channels could as well be characterized by generalizing the  $\Gamma$ -terms to include any spin-selective relaxation channels, while  $\gamma$ -terms to include spin-independent ones.

During the gate operation, the system is initially in the state  $\rho_i = |\Psi_i\rangle\langle\Psi_i| \otimes |0\rangle_a\langle 0|_a$ , and then evolves under equation (2) for a duration of  $t = 2\pi/\Omega$ , resulting in a final density  $\rho_f$ . The fidelity is defined as

$$\mathcal{F} = \text{tr}\{\rho_f \hat{U}_\pi |\Psi_i\rangle_{23} \langle\Psi_i|_{23} \hat{U}_\pi^\dagger \otimes \hat{P}_1\}, \quad (4)$$

where  $\hat{P}_1 = |0\rangle_a\langle 0|_a + |1\rangle_a\langle 1|_a$ . This gives the probability that two logical qubits are in the proper phase-gate output state with the electron-hole pair remaining in QD1. This later condition is required because relaxation of the exciton to either QD2 or QD3 results in a doubly-charged dot, thus spoiling the separability of the two-qubit subsystem.

Before presenting numerical results, we first seek approximate analytical solutions to the dissipative dynamics of equation (2). Defining density matrices  $\rho_{mn} = \langle m|_{23} \hat{P}_1 \rho \hat{P}_1 |n\rangle_{23}$ , with  $m, n = \uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$ , the master equation (2) can be divided into a set of uncoupled equations, leading to

$$\begin{aligned} \frac{\partial\rho_{mn}}{\partial t} = & i[\rho_{mn}, H_0] + \frac{\gamma}{2} \left( c_1\rho_{mn}c_1^\dagger - c_1^\dagger c_1\rho_{mn} + H.c. \right) \\ & - \alpha_m \Gamma c_1^\dagger c_1 \rho_{mn} - \alpha_n \Gamma \rho_{mn} c_1^\dagger c_1, \end{aligned} \quad (5)$$

where  $\alpha_m$  is a logical-qubit dependent parameter, defined as  $\alpha_m = 0, \frac{1}{2}, \frac{1}{2}, 1$ , for  $m = \uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$ , respectively.

Successful operation requires  $\Gamma \gg \Omega$  to impose the QZE, while  $\gamma \ll \Omega$  is required to suppress spontaneous photon-emission, the primary failure mechanism. Hence, the operational range of the present Zeno phase gate is  $\gamma \ll \Omega \ll \Gamma$ . This separation of time-scales enables us

to solve Eq. (5) perturbatively. With the definition  $\rho_{mn}^\lambda = \langle \lambda m | \rho | \lambda n \rangle$ , the matrix elements of the final density are given to second order in  $\frac{\gamma}{\Omega}$  and  $\frac{\Omega}{\Gamma}$  by  $\rho_{mn}^\lambda = \mu_{mn}^\lambda \langle m | \Psi_i \rangle \langle \Psi_i | n \rangle$ . The output coefficients  $\mu_{mn}^\lambda$  are given by Table 1, with  $f(x) = 1 - \frac{\pi}{2}x + \frac{3\pi^2}{50}x^2$ ,  $g(x) = \frac{\pi}{100}x + \frac{3\pi^2}{500}x^2$ . Note that the population and coherence dynamics in the subspace  $\lambda = 2, 3$  are completely decoupled from the  $\lambda = 0, 1$  subspace. In fact, we only need equations for the diagonal matrix elements with respect to the  $\lambda = 0, 1$  subspace, as only they contribute to the fidelity (4). We see from Table 1 that to leading order, the gate output coefficients are consistent with only the state  $|\uparrow\uparrow\rangle_{23}$  having acquired a  $\pi$ -phase shift, as desired for the  $\pi$ -phase gate.

TABLE I: Output coefficients

$m$	$n$	$\mu_{mn}^0$	$\mu_{mn}^1$
$\uparrow\uparrow$	$\uparrow\uparrow$	$(1 + \frac{3\pi}{4}\frac{\gamma}{\Omega})^{-1}$	$1 - (1 + \frac{3\pi}{4}\frac{\gamma}{\Omega})^{-1}$
$\uparrow\uparrow$	$\neq\uparrow\uparrow$	$-f(\frac{\Omega}{\alpha_n\Gamma}) \exp(-\frac{\pi}{2}\frac{\gamma}{\Omega})$	$g(\frac{\Omega}{\alpha_n\Gamma}) (\frac{\gamma}{\Omega} + \pi\frac{\gamma^2}{\Omega^2})$
$\neq\uparrow\uparrow$	$\uparrow\uparrow$	$-f(\frac{\Omega}{\alpha_m\Gamma}) \exp(-\frac{\pi}{2}\frac{\gamma}{\Omega})$	$g(\frac{\Omega}{\alpha_m\Gamma}) (\frac{\gamma}{\Omega} + \pi\frac{\gamma^2}{\Omega^2})$
$\neq\uparrow\uparrow$	$\neq\uparrow\uparrow$	$\exp(-\frac{\pi}{2}\frac{\alpha_m + \alpha_n}{\alpha_m\alpha_n}\frac{\Omega}{\Gamma})$	0

The fidelity defined in Eq. (4) is now explicitly given by

$$\mathcal{F} = \sum_{mn} (-1)^{\delta_{m,\uparrow\uparrow} + \delta_{n,\uparrow\uparrow}} (\mu_{mn}^0 + \mu_{mn}^1) |\langle n | \Psi_i \rangle|^2 |\langle m | \Psi_i \rangle|^2, \quad (6)$$

which is dependent on  $\gamma, \Omega, \Gamma$ , as well as the initial logical state  $|\Psi_i\rangle$ . In practice, while  $\Gamma$  and  $\gamma$  are known parameters for a given QD system,  $|\Psi_i\rangle$  is in general arbitrary, making it impossible to globally optimize  $\Omega$  for all initial states. Nonetheless, for a given  $\Omega$  we can always establish a lower bound for the fidelity. Optimizing  $\Omega$  to maximize the lower limit on the fidelity gives  $\Omega_{opt} = \sqrt{\gamma\Gamma/8}$ , with the optimized fidelity obeying

$$\mathcal{F} \geq \exp \left[ -\frac{10}{3} \sqrt{\frac{\gamma}{\Gamma}} \right], \quad (7)$$

which shows that the way to improve the gate fidelity is to decrease the ratio  $\gamma/\Gamma$ .

Considering recent theoretical calculations, the phonon-assisted transfer rate between two QDs can be as fast as several tens of picoseconds for favorable alignments [16]. The lifetime  $\tau$  of the exciton in (In,As)Ga/GaAs QD is of the order of 1 ns [22], which only marginally meets our operational criteria. Nonetheless,  $\tau$  can be significantly extended by

embedding the QD system into an optical cavity. In fact,  $\tau \sim 10$  ns has been demonstrated in a recent experiment [23]. For accessible parameters of  $\Gamma = 20$  ns $^{-1}$ ,  $\gamma = 0.08$  ns $^{-1}$ , we find  $\Omega_{opt} = 0.45$  ns $^{-1}$  and the lower bound for the fidelity is 0.810. For these parameters, the average fidelity [24, 25] is  $\mathcal{F}_{avg} = 0.85$ . As we will describe, a much higher fidelity can be obtained probabilistically by measuring the final state of the ancillary system to *herald* successful gate operation.

To verify the analytical results, we now solve exactly the dissipative dynamics Eq. (1) via numerical simulations. For comparison, we choose  $\Gamma = 20$  ns $^{-1}$ ,  $\gamma = 0.08$  ns $^{-1}$ ,  $\Omega = 0.45$  ns $^{-1}$  and initial state  $|\Psi_i^0\rangle = \frac{1}{2}(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$ . The dynamics of matrix elements  $\rho_{\uparrow\uparrow,\uparrow\uparrow}^{00}, \rho_{\uparrow\downarrow,\uparrow\uparrow}^{00}$  and  $\rho_{\downarrow\uparrow,\downarrow\downarrow}^{00}, \rho_{\downarrow\downarrow,\downarrow\downarrow}^{00}$  are shown in Fig.2. From the figure we can see that both  $\rho_{\uparrow\uparrow,\uparrow\uparrow}^{00}$  and  $\rho_{\uparrow\downarrow,\uparrow\uparrow}^{00}$  undergo damped oscillation, due to  $\gamma \ll \Omega$ . At the end of the  $2\pi$ -pulse, we find  $\rho_{\uparrow\uparrow,\uparrow\uparrow}^{00} = 0.180$ , and  $\rho_{\uparrow\downarrow,\uparrow\uparrow}^{00} = -0.176$ , compared with 0.176 and  $-0.175$  from the analytical results. In contrast, both  $\rho_{\downarrow\uparrow,\downarrow\downarrow}^{00}$  and  $\rho_{\downarrow\downarrow,\downarrow\downarrow}^{00}$  are shown to be frozen in its initial state, due to QZE since  $\Omega \ll \Gamma$ . The final values of  $\rho_{\uparrow\downarrow,\downarrow\uparrow}^{00}$  and  $\rho_{\downarrow\downarrow,\downarrow\downarrow}^{00}$  are found to be 0.217 and 0.233, in agreement with the analytical results. The fidelity from the numerical simulation is  $\mathcal{F}_e = 0.829$ , which is very close to the predicted value 0.831 from equation (6).

The gate fidelity can be further improved by measuring the final state of the ancillary electron-hole pair, which can ‘herald’ successful gate operation. If it is detected in  $|2\rangle_a, |3\rangle_a$  or  $|1\rangle_a$  states, which correspond to a trion in QD2, QD3, or an exciton in QD1, failure is indicated. Only if the state  $|0\rangle_a$  is obtained, is successful operation a possibility. Its fidelity, in this case is then  $\mathcal{F}_h = \mathcal{F}/(1 - P_f)$ , where  $P_f$  is the failure probability. For the input state  $|\Psi_i^0\rangle$ , this heralded fidelity is  $\mathcal{F}_h = 0.986$ , a significant improvement from the unheralded value of 0.829. Similar improvements are found for other input states. The reason for the large improvement is that the dominant failure mechanism is photon emission via exciton decay in QD1. This is most likely to occur at the halfway point of gate operation. This results in QD1 returning to  $|0\rangle_a$  and begin a new Rabi oscillation cycle. In this scenario, only half of a Rabi cycle will have occurred, leaving QD1 in the exciton states. Thus failure due to photon emission will correlate highly with the ancillary system being found in state  $|1\rangle_a$  at the end of gate operation.

In practice, one feasible way to measure the final state of the ancillary electron-hole pair is to apply two driven lasers to the three QDs and detect the outcoming fluorescence photons. One of the lasers is tuned to be resonant with the trion and charged biexciton



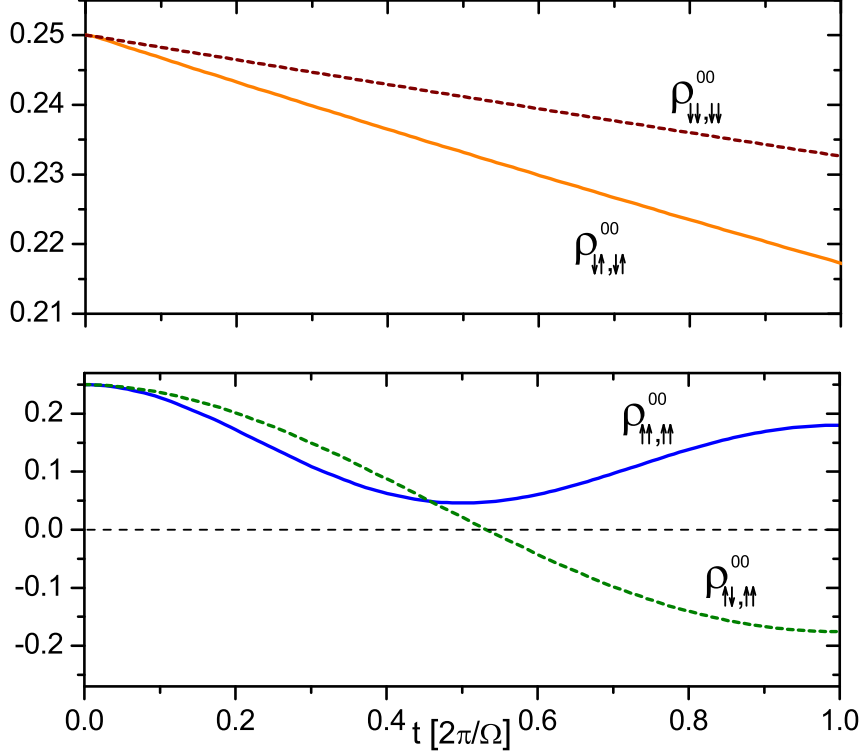


FIG. 2: Dynamical evolution of several density matrix elements for the initial state  $|\Psi_i^0\rangle$  during the gate operation via numerical simulation.

transition in QD2 and QD3, yet far detuned from other transitions. Similarly, the other laser is resonant with the exciton and biexciton transition in QD1, and as well far detuned from other transitions. A trion in QD2 and QD3, or an exciton in QD1, will then lead to resonance fluorescence, indicating the failure of the gate operation. On the other hand, the absence of fluorescence photons heralds the electron-hole being in the  $|0\rangle_a$  state. We note that in the non-fluorescence case, the logical qubits are preserved, since they are only far off-resonantly coupled by the driven fields.

The results of this work demonstrate the possibility to realize a two-qubit controlled phase gate via the QZE in (In,As)Ga/GaAs self-assembled quantum dots. Using experimental values for all parameters, the obtained fidelity around 0.85. If the final state of the exciton can successfully be measured, the heralded fidelity can be as high as 0.99. The fidelity can be improved further only if the phonon-assisted exciton transfer rate can and/or the lifetime of the exciton in the ancillary dot is increased. These might be possible with different materials, such as II-VI based systems.

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